TORONTO METROPOLITAN UNIVERSITY
DEPARTMENT OF COMPUTER SCIENCE

CPS 420 FINAL EXAM
WINTER 2024

INSTRUCTIONS

- This exam is 120 minutes long, but you can all continue writing it for a third hour.
- This exam is out of 60 and is worth 30% of the course mark.
- This is a closed book exam. However, one double-sided letter-sized crib sheet is allowed.
- This exam is double-sided and has 10 pages including this front page. The last 2 pages are blank. Therefore there are 7 pages of questions: pages 2 to 8 inclusive.
- Please answer all questions directly on this exam. If you need extra space to finish answering questions, please do so on the blank pages at the end and indicate very clearly on the original page of each question on which page the rest of your answer can be found.

PART A – REGULAR EXPRESSIONS AND FINITE STATE AUTOMATA – 30 MARKS

A1. Equivalent Automata (8 marks)

a) (3 marks) Draw a **non-deterministic** automaton (NFA) to accept the regular language L(0*(012)*2*) defined over the alphabet {0,1,2}

b) (5 marks) Draw a **deterministic** automaton (DFA) to accept the regular language L(0*(012)*2*) defined over the alphabet {0,1,2}.

A2. Strong passwords (16 Marks)

The topic of this question is related to question B2 of this exam. The two questions are independent of each other and can be answered in any order.

Question A2 consists of two parts: a) on this page and b) on the next page. These questions are related, but it is possible to answer them independently. You should answer them in the order that makes the most sense to you. However, these questions will be graded completely independently of each other, and your grade for each question will not take into account your answers to the other question.

a) (8 marks) A website accepts passwords created with characters from the alphabet {a,b,c,X,Y,Z,1,2,3,#,$}. Valid passwords consist of strings over this alphabet that contain at least 1 digit, at least 1 upper case letter, and at least 1 special character.

Draw a **deterministic** finite automaton (DFA) that accepts valid passwords and rejects invalid ones and has as few states as possible.

A2. Strong passwords - Continued

b) (8 marks) A website accepts passwords created with characters from the alphabet {a,b,c,X,Y,Z,1,2,3,#,$}. Valid passwords consist of strings over this alphabet that contain at least 1 digit, at least 1 upper case letter, and at least 1 special character.

Give a regular expression to describe all valid passwords. **This expression should be as factored as it can be**.
You can use the UNIX regex [] notation if you wish, where [abc] means a|b|c

A3. Canonical orders (6 marks)

Define the two languages L_1 and L_2 over the alphabet Σ={a,b} as: $L_1 = L((ab)^*)$ $L_2 = L(a^*)$
The first five elements in canonical order of $L_1 \cup L_2$ and $L_1 L_2$ are the same: ε, a, aa, ab, aaa.
List the next 8 elements of these two sets **in canonical order**:

$L_1 \cup L_2$		$L_1 L_2$	
6:	10:	6:	10:
7:	11:	7:	11:
8:	12:	8:	12:
9:	13:	9:	13:

PART B – COUNTING AND PROBABILITIES – 30 MARKS

In this entire section, you should

- **explain your answers.** Answers without explanations will not get full grades.
- **simplify your calculations** as much as possible, but it is not necessarily to calculate the exact answer. For example, answers consisting of products of powers of integers are fine. Answers can be fractions but the fractions should also be simplified as much as possible.

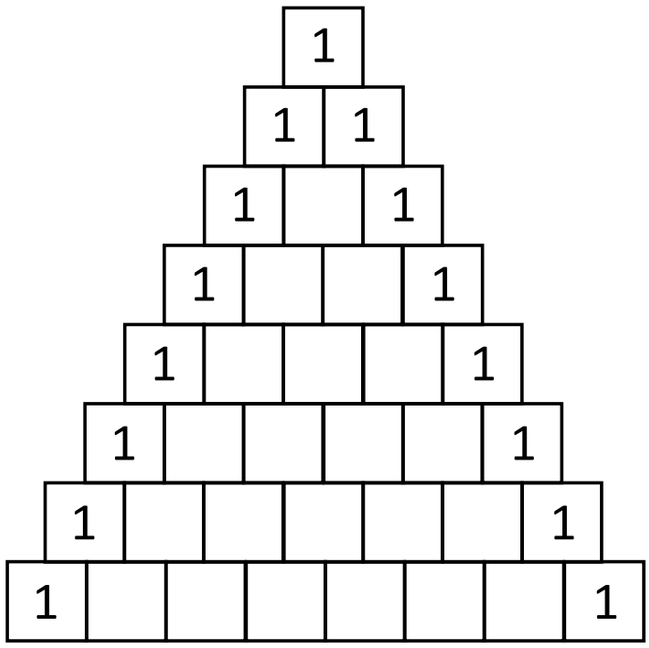
B1. Fibonacci and Pascal (5 marks)

Recall the recursive definition of the Fibonacci sequence: $\begin{cases} F_0 = 1, F_1 = 1 \\ F_n = F_{n-1} + F_{n-2} \text{ for } n \geq 2 \end{cases}$

It turns out that $F_n = \sum_{i=0}^{\lfloor n/2 \rfloor} \binom{n-i}{i}$ where $\lfloor x \rfloor$ denotes the floor of x.

Therefore the Pascal triangle can be used to calculate the values of the Fibonacci sequence.

Draw and **use the Pascal triangle (and not the recursive definition of F or the formula for $\binom{n}{k}$)** to calculate F_7.



B2. Strong Passwords (6 marks)

The topic of this question is related to question A2 of this exam. The two questions are independent of each other and can be answered in any order.

A web site mandates that passwords for its accounts be 8-character long with the characters taken from the set consisting of all 10 digits, the 52 upper and lower-case letters used in the English alphabet and 10 special characters.

a) (2 marks) Unless coerced to do otherwise, many users will simply create a password with only lower-case letters. What is the probability that a hacker can break into an account protected with such a password by trying to log into it with a randomly generated password of 8 lower-case letters?

b) (4 marks) If the web site mandates that all passwords include **at least** one digit, at least one upper case character, and at least one special character, does that improve or worsen an account's security compared with the type of password created in a), and by how much?

B3. Investment Funds (10 marks)

a) (3 marks) How many different ways can you invest $20,000 into six different funds in increments of $1000? For example, one way to do this is [$0 ; $4,000 ; $1,000 ; $2,000 ; $11,000 ; $2,000]

b) (3 marks) You really don't know anything about these six funds, so for each of your twenty $1,000 you will roll a die to pick the fund to invest it in. What is the probability that you will invest some money in all six funds?

c) (4 marks) You did end up investing money in all six funds. What is the probability that you invested exactly $3,000 in the third fund?

B4 Rolling dice (4 marks)

In this question you will be rolling three fair 6-sided dice with sides {1,2,3,4,5,6} at the same time and looking at the total of the three values rolled.
How many times **must** you roll three dice **to be sure** to obtain the same total from the three dice at least six times?
Hint: this is a pigeonhole question.

B5. Snowy Roads (5 marks)

There are two roads from A to B and two roads from B to C. Each of the four roads has probability *p* of being blocked by snow, independently of all the others. What is the probability that there is an open road from A to C?

THIS PAGE IS INTENTIONALLY LEFT BLANK AND CAN BE USED FOR ROUGH WORK OR TO CONTINUE ANSWERING AN EARLIER QUESTION.

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