

**TORONTO METROPOLITAN UNIVERSITY
DEPARTMENT OF COMPUTER SCIENCE**

**CPS 420
FINAL EXAM
WINTER 2023**

INSTRUCTIONS

- This exam is 120 minutes long, but you can all continue writing it for a third hour.
- This exam is out of 60 and is worth 30% of the course mark.
- This is a closed book exam. However, one double-sided letter-sized crib sheet is allowed.
- This exam is double-sided and has 10 pages including this front page. The last 3 pages are blank. Therefore there are 6 pages of questions: pages 2 to 7 inclusive.
- Please answer all questions directly on this exam. If you need extra space to finish answering questions, please do so on pages 8 to 10 and indicate very clearly on the original page of each question on which page the rest of your answer can be found.

PART A – REGULAR EXPRESSIONS AND FINITE STATE AUTOMATA – 30 MARKS

A1. Operations on Languages (6 marks)

Let the following two languages L_1 and L_2 over the alphabet $\Sigma=\{0,1\}$ be defined as:

$L_1 = \{\epsilon, 1, 00\}$ $L_2 = \{0, 1, 11\}$

a) (1 mark) List all the elements of $L_1 \cup L_2$

b) (1 mark) List all the elements of $L_1 \cap L_2$

c) (2 marks) List all the elements of $L_1 \times L_2$

d) (2 marks) List all the elements of $L_1 L_2$

A2. Understanding Automata (6 marks)

In each empty box below, give a regular expression describing the language accepted the automaton above it. Make these regular expressions as simple as possible.

A3. Problem solving with FAs (18 Marks) – Continued on next page

Given the alphabet $\Sigma = \{f, o, d, a\}$, define the following language L over Σ^* :

L is the set of all words ending in a vowel (o or a), that start with a consonant (f or d) followed by a vowel, and have these additional properties:

- Two vowels are never next to each other
- The only way two consonants can be next to each other is when a consonant is doubled (but not tripled).

Please note that all the Q3 questions on this page and the next one are related. They have been ordered in an order that facilitates answering them, but it is possible to answer them independently. Therefore you can answer these questions in the order that makes the most sense to you.

a) (3 marks) Give a regular expression to describe L

A3. Continued

- b) (5 marks) Draw a non-deterministic finite automaton (NFA) to accept L
- c) (7 marks) Draw a **complete** deterministic finite automaton (DFA) to accept L. (In a complete DFA, all possible strings of input are handled)
- d) (3 marks) Draw a deterministic finite automaton (DFA) to accept the language $\Sigma^* - L$

PART B – COUNTING AND PROBABILITIES – 30 MARKS

In this entire section, you should simplify your calculations as much as possible. Final answers can be fractions but they should also be simplified as much as possible.

B1 Taking the bus (8 marks)

20 people are lining up to get on a bus in random order. 10 will pay an adult fare, 6 a youth fare, and 4 a senior fare.

a) (4 marks) In how many different orders can the 20 fares be paid when these people board the bus? Explain your answer.

b) (4 marks) What is the probability that the third person to board the bus will pay a youth fare? Explain your answer.

B2 Valentine (7 marks)

You are buying a box of 12 chocolate truffles as a Valentine's day present. The chocolate store has a very large supply of 10 different varieties of truffles. Because Valentine's day is one of the busiest days in the chocolate world, they only offer prepackaged boxes where all the 12 truffles have been randomly selected from the 10 varieties. Your Valentine's favourite truffle flavor is lemon, which is one of the 10 varieties that the store carries.

What is the probability that the box you are purchasing will contain at least one lemon truffle? Explain your answer.

B3 Bingo (5 marks)

A bingo ball contains 75 tokens numbered from 1 to 75. The game host draws a token randomly from the ball but simply says that the number is a multiple of 8. What is the probability that the number drawn contains the digit 4? Explain your answer.

B4 Jelly Beans (5 marks)

You are offered jelly beans from a jar that contains 50 jelly beans: 20 orange-flavoured (O), 15 lime-flavoured (L), and 15 cherry-flavoured (C). The jelly beans are very well mixed up in the jar. You dip your hand in the jar and grab 6 jelly beans without looking.

What is the expected value of the number of cherry-flavoured jelly beans in your hand? Explain your answer.

B5 Binomial Theorem (5 marks)

Use the binomial theorem to give a full expansion of $(3x-2)^4$. **Show your intermediate steps and be careful with the negative signs.**

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