RYERSON UNIVERSITY
DEPARTMENT OF COMPUTER SCIENCE

CPS 420
MIDTERM 1
WINTER 2020

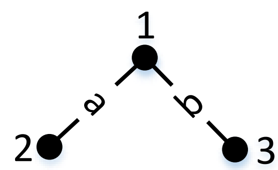
INSTRUCTIONS

- This exam is 120 minutes long.
- This exam is out of 50 and is worth 15% of the course mark.
- This is a closed book exam. However, one double-sided letter-sized crib sheet is allowed.
- This exam is double-sided and has 8 pages including this front page. The last two pages are blank. Therefore there are 5 pages of questions: pages 2 to 6 inclusive.
- Please answer all questions directly on this exam. If you need extra space to finish answering questions, please do so on pages 7 and 8 and indicate very clearly on the original page of each question on which page the rest of your answer can be found.

PART A – GRAPH THEORY – 20 MARKS

1. <u>Subgraphs and Connectivity (10 marks)</u>

Draw all the possible subgraphs of G in the boxes below, and for each subgraph indicate whether it is connected or disconnected by circling Y (if connected) or N (if disconnected).
Note that the number of boxes may not match the number of possible subgraphs.
G is the following graph:

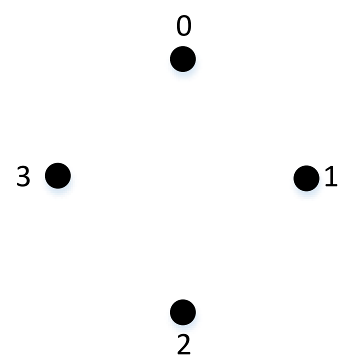


Connected: Y N	Connected: Y N	Connected: Y N	Connected: Y N
Connected: Y N	Connected: Y N	Connected: Y N	Connected: Y N
Connected: Y N	Connected: Y N	Connected: Y N	Connected: Y N
Connected: Y N	Connected: Y N	Connected: Y N	Connected: Y N

2. Matrices in Graph Theory (10 marks)

a) On the diagram on the right, draw the directed graph G described by the adjacency matrix on the left (i.e. add the edges in G)

	0	1	2	3
0	0	0	1	0
1	0	1	1	0
2	1	0	1	0
3	1	0	0	0



b) Fill out the matrix A below which is defined as follows:

A(i,j) = number of walks of length 2 from vertex i to vertex j in the graph G.

i \ j	0	1	2	3
0				
1				
2				
3				

PART B – SEQUENCES, RECURRENCE RELATIONS – 10 MARKS

Given the sequence a_n defined with the recurrence relation:

$a_0 = 5$

$a_k = a_{k-1} + k + 2^k$ for $k \geq 1$

1. Terms of the Sequence (4 marks)

Calculate a_1, a_2, a_3, a_4
Keep your intermediate answers as you may need them in the next question.

2. Iteration (6 marks)

Using iteration, solve the recurrence relation when $n \geq 0$ (i.e. find an analytic formula for a_n). Simplify your answer as much as possible, showing your work. In particular, your final answer should not contain sums (Σ) and products ($\prod$)

PART C – INDUCTION – 20 MARKS

In this question you will prove by mathematical(weak) induction the following theorem:

for any positive integer n, $1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$

Before you start you will need to translate this theorem in symbolic form, in the form of
$\forall n \in D$, P(n)

1. Set D (1 mark)

What is the set D in the symbolic form $\forall n \in D$, P(n) of the theorem you will prove?

2. P(n) (4 marks)

What is the predicate function P(n) in the symbolic form $\forall n \in D$, P(n) of the theorem you will prove? You will need to use Σ notation in this definition of P(n).

You will now prove the theorem by mathematical induction. No other method is acceptable. Be sure to lay out your proof clearly and correctly and to justify every step.

3. Basic Step of the Proof (4 marks)

Write the basic step of your proof here.

4. Inductive Step of the Proof (11 marks)

Write the inductive step of your proof here.

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WORK ON THIS PAGE WILL ONLY BE GRADED IF SPECIFICALLY REQUESTED ON ONE OF PAGES 2 TO 6.

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