RYERSON UNIVERSITY
DEPARTMENT OF COMPUTER SCIENCE

CPS 420
MIDTERM 1
WINTER 2019

INSTRUCTIONS

- This exam is 120 minutes long.
- This exam is out of 50 and is worth 15% of the course mark.
- This is a closed book exam. However, one double-sided letter-sized crib sheet is allowed.
- This exam is double-sided and has 8 pages including this front page. The last two pages are blank. Therefore there are 5 pages of questions: pages 2 to 6 inclusive.
- Please answer all questions directly on this exam. If you need extra space to finish answering questions, please do so on pages 7 and 8 and indicate very clearly on the original page of each question on which page the rest of your answer can be found.

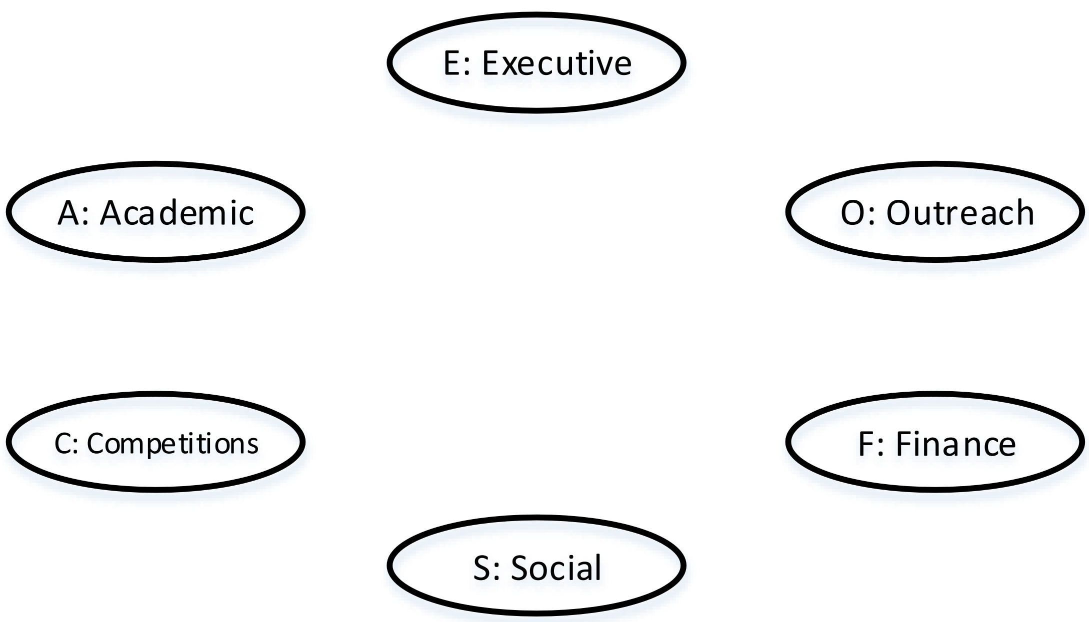
PART A – GRAPH THEORY – 20 MARKS

In this questions a graph will be used to solve a scheduling problem:
12 course union reps have to serve on the following 6 committees:

- E: Executive: Lina, Mohammed, Taz, Seden, Maryam
- A: Academic: Maryam, Nanh, Rohan, Lina
- C: Competitions: Zixuan, Christina, Luigi
- S : Social: Taz, Christina, Daniel
- F: Finance: Mohammed, Rohan, Zixuan
- O: Outreach and careers: Alexandra, Nanh, Daniel

1. <u>Committee Overlaps (4 marks)</u>

Draw the edges of the graph below to represents the following relationship between the committees: there is an edge between two different committees if they have at least one member in common:

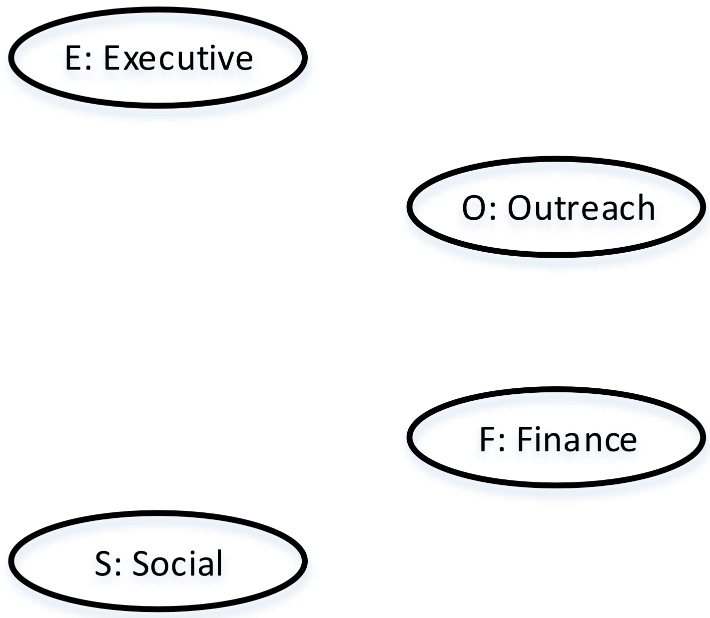


2. <u>Committee Meetings (6 marks)</u>

The committees meet every week, but the student members only have three free time slots during the week: from 12 to 1 on Mondays (M), Wednesdays(W) or Fridays(F). Label each vertex in the graph above with one of the letters M, W, or F to indicate on which of the three days the committee represented by the vertex should meet to make sure that all the student reps can attend all the meetings of all the committees on which they are serving. In other words, two adjacent committees should meet on different days so that the members they have in common can attend both meetings. (This question has more than one correct answer).

3. Modified Committee Overlaps (2 marks)

Seden has now replaced Luigi on the Competitions committee. Taz, who is thinking of running for president next year, has also volunteered to serve on the Finance Committee. Redraw the committee overlaps graph to reflect these changes in membership. For this question, you do not need to label the vertices with M, W, or F.



4. Cliques (8 marks)

Given a graph G with n vertices, and an integer k≤n, a k-clique of G is a complete subgraph of G with k vertices.

a) List all the 3-cliques of the graph in question 3 above. Describe each 3-clique as a set of vertices.

b) List all the 4-cliques of the graph in question 3 above. Describe each 4-clique as a set of vertices

5. Further Thoughts (3 bonus marks)

Only attempt this question if you have any remaining time at the end of the exam.
Would it now be possible to schedule meetings for all of these committees in the three available time slots in such a way that all the student reps could attend all the meetings of the committees on which they are serving? Explain your answer.

PART B – SEQUENCES, RECURRENCE RELATIONS – 10 MARKS

Given the sequence a_n defined with the recurrence relation:

$a_0 = 2$

$a_k = 4k + a_{k-1} + 2$ for $k \geq 1$

1. Terms of the Sequence (4 marks)

Calculate a_1, a_2, a_3, a_4

Keep your intermediate answers as you may need them in the next question.

2. Iteration (6 marks)

Using iteration, solve the recurrence relation when $n \geq 0$ (i.e. find an analytic formula for a_n). Simplify your answer as much as possible, showing your work. In particular, your final answer should not contain sums (Σ) and products ($\prod$)

PART C – INDUCTION – 20 MARKS

In this question you will prove by strong induction the following theorem:
for any positive integer n, the units digit of 4^n is either 4 or 6.

Before you start you will need to translate this theorem in symbolic form, in the form of $\forall n \in D$, P(n)

1. Set D (1 mark)

What is the set D in the symbolic form $\forall n \in D$, P(n) of the theorem you will prove?

2. P(n) (4 marks)

What is the predicate function P(n) in the symbolic form $\forall n \in D$, P(n) of the theorem you will prove?

You will now prove the theorem by strong induction No other method is acceptable.
Be sure to lay out your proof clearly and correctly and to justify every step.

3. Basic Step of the Proof (4 marks)

Write the basic step of your proof here.

4. <u>Inductive Step of the Proof (11 marks)</u>

Write the inductive step of your proof here.

THIS PAGE IS INTENTIONALLY LEFT BLANK AND CAN BE USED FOR ROUGH WORK OR TO CONTINUE ANSWERING AN EARLIER QUESTION.

WORK ON THIS PAGE WILL ONLY BE GRADED IF SPECIFICALLY REQUESTED ON ONE OF PAGES 2 TO 6.

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