RYERSON UNIVERSITY
DEPARTMENT OF COMPUTER SCIENCE

CPS 420
MIDTERM 2
WINTER 2018

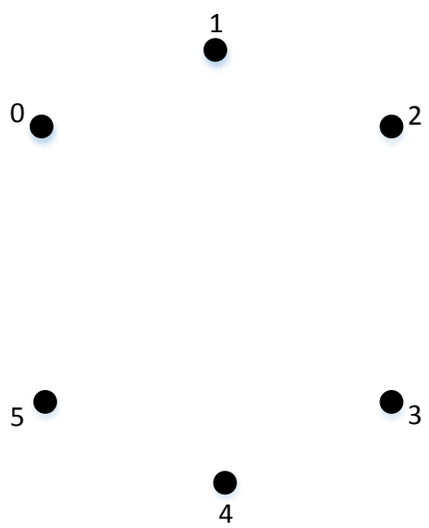
INSTRUCTIONS

- This exam is 120 minutes long.
- This exam is out of 60 and is worth 15% of the course mark.
- This is a closed book exam. However, one double-sided letter-sized crib sheet is allowed.
- This exam is double-sided and has 8 pages including this front page. The last three pages are blank. Therefore there are 4 pages of questions: pages 2 to 5 inclusive.
- Please answer all questions directly on this exam. If you need extra space to finish answering questions, please do so on pages 6 to 8 and indicate very clearly on the original page of each question on which page the rest of your answer can be found.

PART A – GRAPH THEORY – 20 MARKS

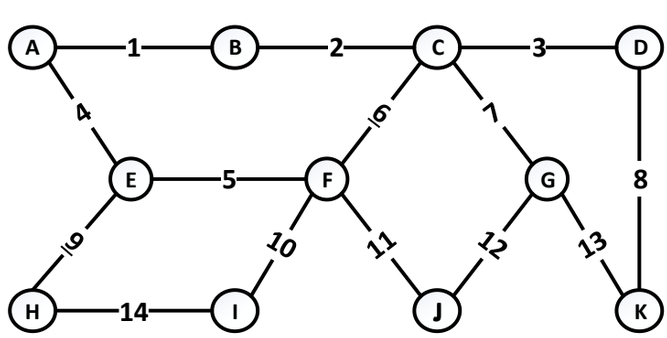
1. Graph of a Relation (4 marks)

Draw the directed graph of the following relation R in the set of vertices S={0,1,2,3,4,5}
$\forall x,y \in S$ xRy iff x mod 3 < y mod 3



2. Circuits (6 marks)

This question is based on the following graph G (the edge numbers are edge names):

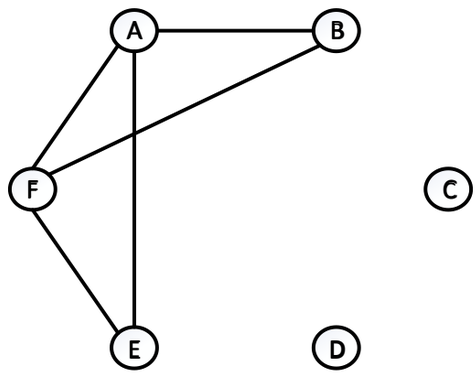


a) Starting at vertex A, give an Euler circuit for G (listing the **vertices and edges** as they are traversed) or explain why this cannot be done

b) Starting at vertex A, give a Hamiltonian circuit for G (listing the **vertices and edges** as they are traversed) or explain why this cannot be done.

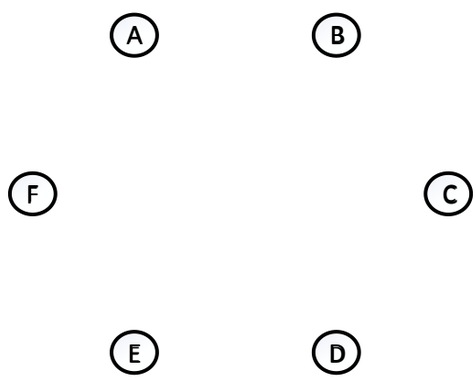
3. Connectedness and Complements (10 marks)

This question is based on the following graph G:



a) List all the connected components of G. Each connected component should be described as the set of all the vertices in the connected component.

b) Draw the complement G^c of the graph G



c) Using the same format as in a) list all the connected components of G^c

PART B – REGULAR EXPRESSIONS AND FINITE STATE AUTOMATA – 40 MARKS

1. Operations on Languages (10 marks)

Define the following two languages of the alphabet $\Sigma=\{0,1,2\}$:
$L_1 = \{0, 01, 02\}$
$L_2 = \{\varepsilon, 2, 02\}$

a) List all the elements of $L_1 \cap L_2$

{ }

b) List all the elements of $L_1 \cup L_2$

{ }

c) List all the elements of $L_1 \times L_2$

{

}

d) List all the elements of $L_1 L_2$

{

}

2. Regular Expression (10 marks)

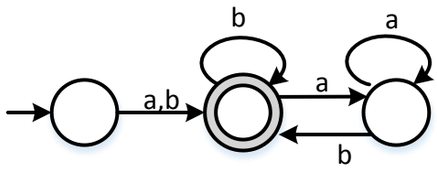
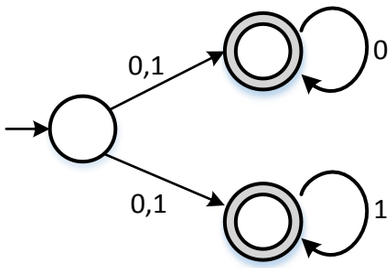
Write a regular expression to match all *sets* in a new programming language. *Sets* are strings like "{}", "{740}", "{hello,799,0,55,friend}" and they are defined as follows:

- A *set* is a list of zero of more *entries* surrounded by curly parentheses.
- If the list contains more than 1 *entry*, the *entries* are separated by commas.
- An *entry* is either a *name* or an *integer*
- A *name* is a string of 1 or more lower-case letter (i.e. a to z)
- An *integer* is either the digit 0 or a string of one or more digits which does not start with the digit 0

You do **not** need to simplify your regular expression

3. Finite State Automata (20 Marks)

a) Give a regular expression for each of the following finite state automata. Make these regular expressions as simple as possible.

Automaton	Regular expression
	
	

In the next two questions the simplest possible automaton refers to an automaton with as few states as possible.

b) Draw the simplest possible NFA (**non-deterministic** finite state automaton) on an input alphabet I={a,b,c} which recognizes the following regular expression: (a|b)(a|c)*(b|c)

c) Draw the simplest possible DFA (**deterministic** finite state automaton) on an input alphabet I={a,b,c} which recognizes the following regular expression: (a|b)(a|c)*(b|c). Your DFA should handle all possible inputs.

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WORK ON THIS PAGE WILL ONLY BE GRADED IF SPECIFICALLY REQUESTED ON ONE OF PAGES 2 TO 5.

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