RYERSON UNIVERSITY
DEPARTMENT OF COMPUTER SCIENCE

CPS 420
MIDTERM 1
WINTER 2018

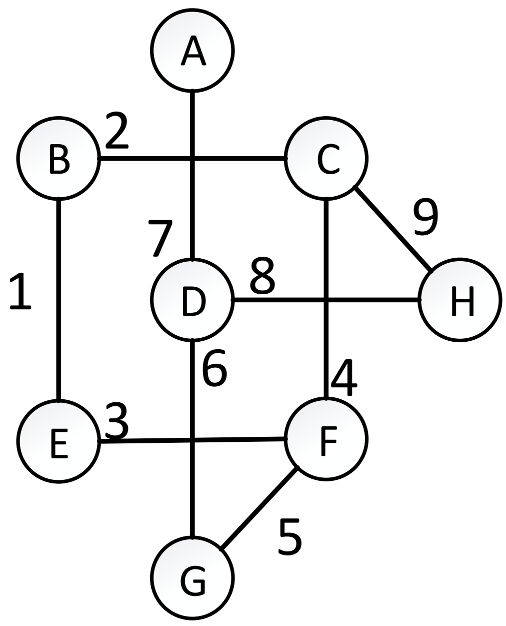
INSTRUCTIONS

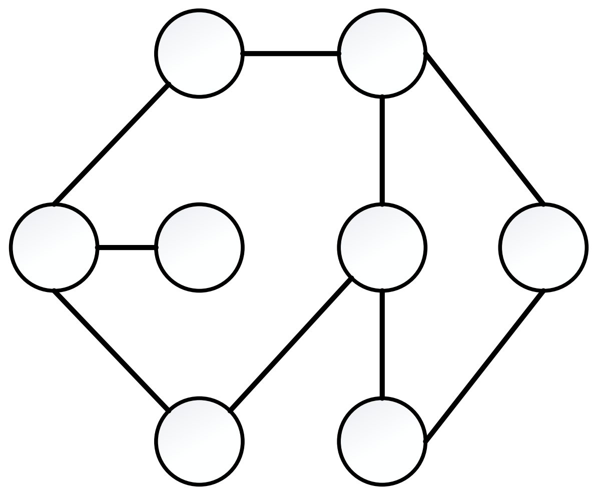
- This exam is 110 minutes long.
- This exam is out of 50 and is worth 15% of the course mark.
- This is a closed book exam. However, one double-sided letter-sized crib sheet is allowed.
- This exam is double-sided and has 8 pages including this front page. The last two pages are blank. Therefore there are 5 pages of questions: pages 2 to 6 inclusive.
- Please answer all questions directly on this exam. If you need extra space to finish answering questions, please do so on pages 7 and 8 and indicate very clearly on the original page of each question on which page the rest of your answer can be found.

PART A – GRAPH THEORY – 20 MARKS

1. <u>Equivalent Graphs (4 marks)</u>

Label the vertices from A to H and edges from 1 to 9 of the graph on the right to show that it is equivalent to the graph on the left





2. <u>Graph Degrees (6 marks)</u>

For each of the following questions, either draw a graph with the requested properties, or explain **convincingly** (possibly by quoting a theorem) why such a graph cannot be drawn.

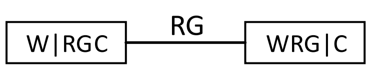
a) A graph with 5 vertices of degrees 5, 5, 4, 4, 3	b) A graph with 5 vertices of degrees 5, 5, 4, 4, 4

3. Problem Solving with Graphs (10 marks)

A rower has to row a box of cabbages, a goat, and a wolf from the left side of a river to the right side. If they are left unsupervised, the goat will eat the cabbages and the wolf will eat the goat. The boat can only carry the rower and at most one of the three items.

Draw a graph showing all the possible legal moves in this problem. In other words, your graph does not need to show actions and states which lead to the destruction of one of the items.

The vertices in your graph show all the valid states, and the edges are transitions that show river crossings. Use the following symbols: R=rower, W=wolf, G=goat, C=cabbages, | = river. There is no need to represent the boat because it will always be with the rower. For example:



Shows the rower rowing the goat across the river when the wolf is on the left bank and the cabbages are on the right bank.

PART B – SEQUENCES, RECURRENCE RELATIONS – 10 MARKS

Given the sequence a_n defined with the recurrence relation:

$a_1 = 1/2$

$a_k = a_{k-1} + \frac{1}{k(k+1)}$ for $k \geq 2$

1. <u>Terms of a Sequence (6 marks)</u>

Calculate a_2, a_3, a_4
Keep your intermediate answers as you may need them in the next question.

2. <u>Iteration (4 marks)</u>

Using iteration, solve the recurrence relation when $n \geq 1$ (i.e. find an analytic formula for a_n). Simplify your answer as much as possible, showing your work. In particular, your final answer should not contain sums (Σ) and products ($\prod$)

PART C – INDUCTION – 20 MARKS

1. Mathematical (weak) Induction (10 marks)

Prove by induction that for all positive integers n, $\sum_{i=2}^{n} i(i-1) = \frac{n(n-1)(n+1)}{3}$
No other method is acceptable.
Be sure to lay out your proof clearly and correctly and to justify every step.

2. Types of induction (10 marks)

Fill out the table below indicating whether each of the proofs by induction described **requires** strong induction. Explain your answers in a convincing way.

Proof description	**Needs** strong induction? (Y/N)	Explanation of your answer
Proof of the correctness of the solution for the sequence a_n recursively defined as: $a_1=2$, $a_2=3$, $a_n=a_{n-2}+2$ for $n>2$		
Proof of the correctness of the solution for the sequence b_n recursively defined as: $b_1=2$, $b_2=5$, $b_n=b_{n-1}+3$ for $n>2$		
Proof that every positive integer has a unique binary representation starting with a leading 1.		

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WORK ON THIS PAGE WILL ONLY BE GRADED IF SPECIFICALLY REQUESTED ON ONE OF PAGES 2 TO 6.

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