RYERSON UNIVERSITY

DEPARTMENT OF COMPUTER SCIENCE

CPS 420
MIDTERM 2
WINTER 2017

NAME: ______________________________

STUDENT ID: ______________________________

INSTRUCTIONS

- This exam is 110 minutes long.
- This exam is out of 60 and is worth 15% of the course mark.
- This is a closed book exam. However, one double-sided letter-sized crib sheet is allowed.
- This exam is single-sided and has 5 pages including this front page.
- Please answer all questions directly on this exam.

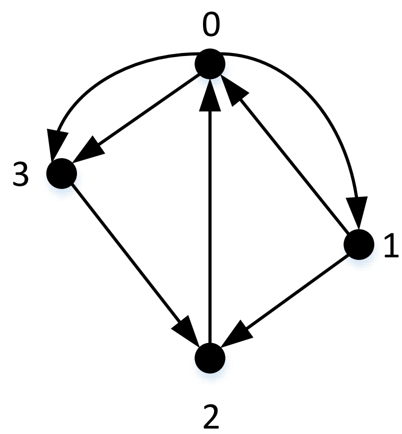
For Grading Purposes

A1-2	**/10**
A3-4	**/10**
B1-2	**/20**
B3	**/20**
TOTAL	**/60**

PART A – GRAPH THEORY – 20 MARKS

1. Matrices in Graph Theory (10 marks)

   This question is based on the following graph G:



a) Fill out the adjacency matrix for the graph G:

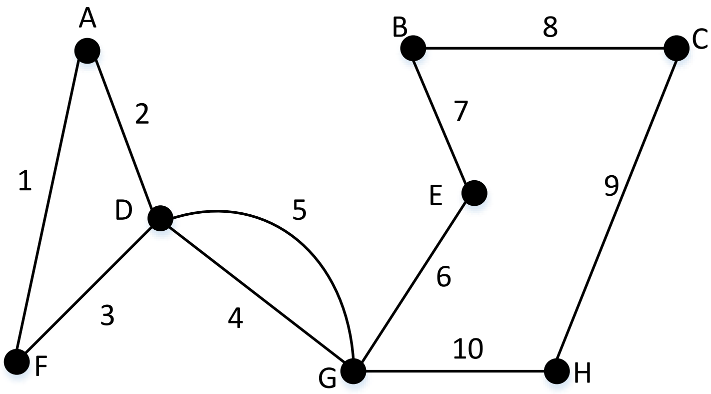
	0	1	2	3
0				
1				
2				
3				

b) Fill out the following matrix A which is defined as follows:
A(i,j) = number of walks of length 2 from vertex i to vertex j in the graph G.

	0	1	2	3
0				
1				
2				
3				

2. Circuits (6 marks)

This question is based on the following graph G (the edge numbers are edge names):

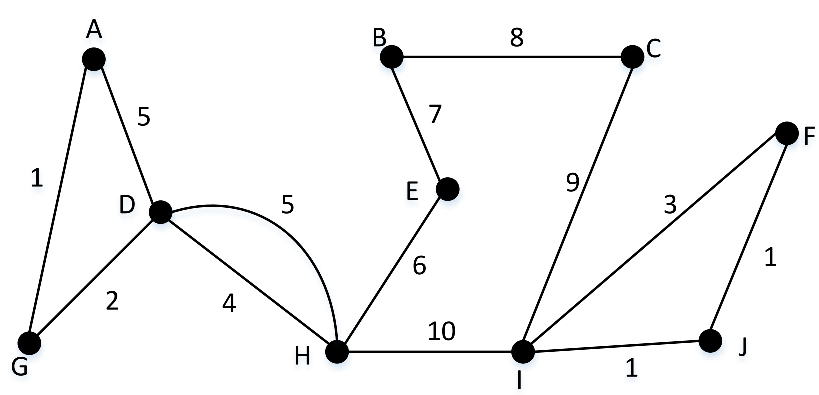


a) Give an Euler circuit for G (listing the vertices and edges as they are traversed) or explain why this cannot be done

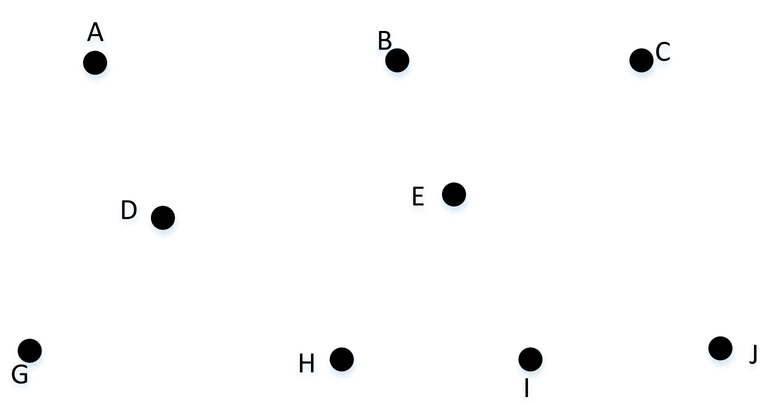
b) Give a Hamiltonian circuit for G (listing the vertices and edges as they are traversed) or explain why this cannot be done.

3. Minimum Spanning Tree (4 marks)

For the weighted graph G underneath, where the edge numbers are weights:



Draw a minimum spanning tree (draw the edges you are keeping with their weights)



PART B – REGULAR EXPRESSIONS AND FINITE STATE AUTOMATA – 40 MARKS

1. Operations on Languages (10 marks)

Define the following two languages of the alphabet $\Sigma=\{a,b\}$:
$L_1 = \{a, aa, ab\}$
$L_2 = \{b, bb, ab\}$

a) List all the elements of $L_1 \cap L_2$

$\{ \qquad \}$

b) List all the elements of $L_1 \cup L_2$

$\{ \qquad \}$

c) List all the elements of $L_1 \times L_2$

$\{ \qquad \}$

d) List all the elements of $L_1 L_2$

$\{ \qquad \}$

2. Regular Expression (10 marks)

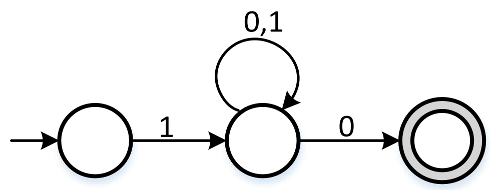
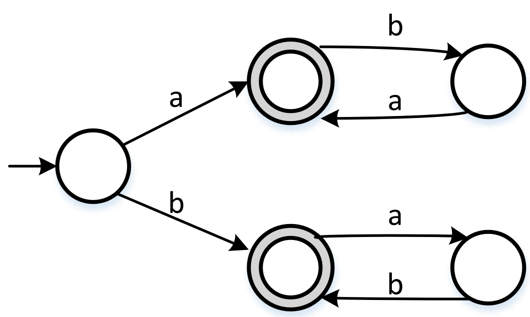
Write a regular expression to match all integers in a new programming language. Integers are defined as follows:

- An integer can be in base ten (decimal), base sixteen (hex) of base eight (octal).
- A decimal integer is either the single digit 0 or a sequence of one or more digits between 0 and 9 such that the leading digit is not a 0.
- A hex number starts with the string “0x” which is then followed by one or more digits between 0 and 9 or letters between A and F.
- An octal number starts with the single digit 0 which is then followed by one or more digits between 0 and 7.

You do **not** need to simplify your regular expression

3. Finite State Automata (20 Marks)

a) Give a regular expression for each of the following finite state automata. Make these regular expressions as simple as possible.

Automaton	Regular expression
	
	

In the next two questions the simplest possible automaton refers to an automaton with as few states as possible.

b) Draw the simplest possible NFA (non-deterministic finite state automaton) on an input alphabet I={a,b,c} which recognizes the following regular expression: (a|b)*abc(b|c)*

c) Draw the simplest possible DFA (deterministic finite state automaton) on an input alphabet I={a,b,c} which recognizes the following regular expression: (a|b)*abc(b|c)*