# RYERSON UNIVERSITY

## DEPARTMENT OF COMPUTER SCIENCE

## CPS 420
## MIDTERM 1
## WINTER 2017

NAME: ______________________________________________

STUDENT ID: ______________________________________________

### INSTRUCTIONS

- This exam is 110 minutes long.
- This exam is out of 60 and is worth 15% of the course mark.
- This is a closed book exam. However, one double-sided letter-sized crib sheet is allowed.
- This exam is single-sided and has 5 pages including this front page.
- Please answer all questions directly on this exam.

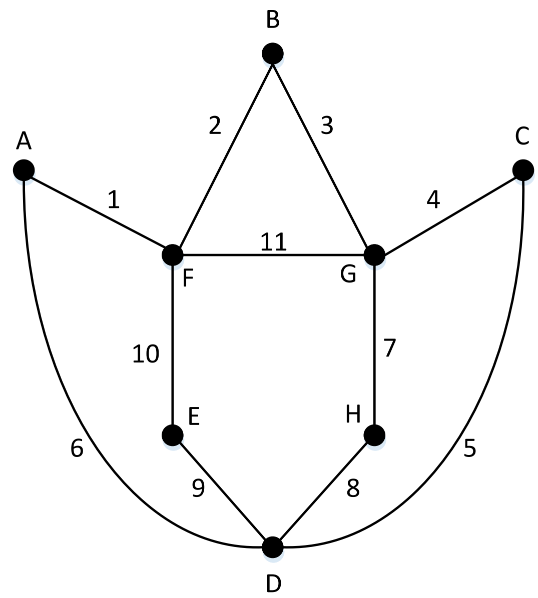
For Grading Purposes

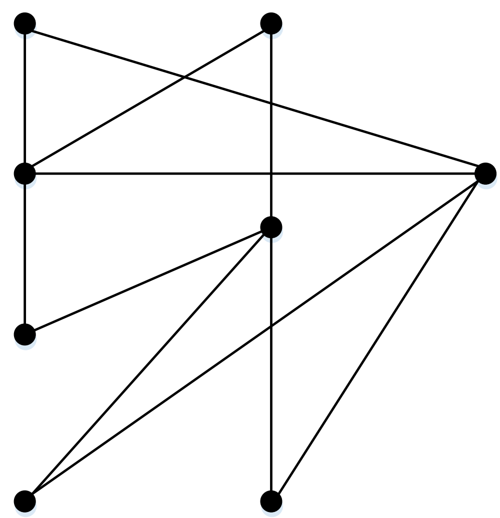
| | |
|---|---|
| **A1-2** | **/10** |
| **A3-5** | **/20** |
| **B** | **/10** |
| **C** | **/20** |

## PART A – GRAPH THEORY – 30 MARKS

1. Equivalent Graphs (4 marks)

Label the vertices from A to H and edges from 1 to 11 of the graph on the right to show that it is equivalent to the graph on the left



2. Graph Degrees (6 marks)

For each of the following questions, either draw a graph with the requested properties, or explain **convincingly** (possibly by quoting a theorem) why such a graph cannot be drawn.

| | |
|---|---|
| a) A graph with 5 vertices of degrees 5, 5, 4, 4, 3 | b) A graph with 5 vertices of degrees 5, 5, 4, 4, 4 |

3. Acquaintance Graphs (8 marks)

Suppose that in a group of 6 people A, B, C, D, E, and F the following pairs of people are acquainted with each other: A and B, B and D, A and D, A and F, C and D, C and E, C and F.

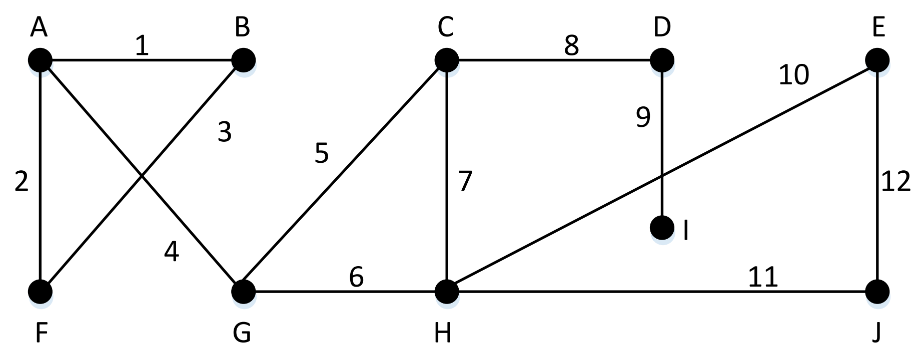
a) Draw a graph G to represent who **is** acquainted with whom

b) Draw a graph H to represent who **is not** acquainted with whom

c) What is G ∪ H?

4. Connected Components (8 marks)

This question and the next one are based on this graph:




a) An edge of a graph whose removal disconnects the graph of which it is a part is called a "bridge". List all the bridges of the graph above.

b) If you were to remove all the bridges that you listed in part a) from the graph, how many connected components would this graph have? List them.

5. Walks (4 marks)

For each of the 4 walks in the graph in question 4, Indicate with True of False in the table below whether the walks in the graph in question 4 have each of the properties

| Walk: | A1B3F2A4G | H6G5C7H11J12E10H | C8D8C7H6G5C | I9D8C5G4A |
|---|---|---|---|---|
| Path/Trail | | | | |
| (Simple) path | | | | |
| Closed walk | | | | |
| Circuit | | | | |
| Simple circuit | | | | |

## PART B – SEQUENCES AND RECURRENCE RELATIONS – 10 MARKS

Given the sequence $a_n$ defined with the recurrence relation:

$a_1 = 1$

$a_n = a_{n-1} + n + 1$ for $n>1$

1. Terms of a Sequence (5 marks)

Calculate $a_2$, $a_3$, $a_4$, $a_5$, $a_6$
Keep your intermediate answers as you will need them in the next question.

2. Iteration (5 marks)

Using iteration, solve the recurrence relation when $n \geq 1$ (i.e. find an analytic formula for $a_n$). Simplify your answer as much as possible, showing your work. In particular, your final answer should not contain sums and products.

## PART C – INDUCTION – 20 MARKS

Prove by induction that for all positive integers n, $\sum_{i=2}^{n} i(i-1) = \frac{n(n-1)(n+1)}{3}$
No other method is acceptable.
Be sure to lay out your proof clearly and correctly and to justify every step.